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Pricing Mortgage Insurance with Asymmetric Jump Risk and Default Risk: Evidence in the U.S. Housing Market

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Abstract This study provides the valuation of mortgage insurance (MI) considering upward and downward jumps in housing prices, which display separate distributions and probabilities of occurrence, and the mortgage insurer's default risk. The empirical results indicate that the asymmetric double exponential jump diffusion performs better than the log-normally distributed jump diffusion and the Black-Scholes model, generally used in previous literature, to fit the single-family mortgage national average of all home prices in the US. Finally, the sensitivity analysis shows that the MI premium is an increasing function of the normal volatility, the mean down-jump magnitudes, the shock frequency of the abnormal bad events, and the asset-liability structure of the mortgage insurer. In particular, the shock frequency of the abnormal bad events has the largest effect of all parameters on the MI premium. The asset-liability structure of the mortgage insurer and shock frequency of the abnormal bad events have a larger effect of all parameters on the default risk premium.

Keywords Mortgage insurance contract · Asymmetric double exponential jump diffusion process · Default risk

JEL Classification G1 · G2

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Introduction

Private mortgage insurance (MI) guarantees that if a borrower defaults on a loan, a mortgage insurer will pay the mortgage lender for any loss resulting from a property foreclosure up to 20–30% of the claim amount. Many previous studies indicate that the housing price change is a crucial factor in determining MI premiums (e.g., Kau et al. 1992, 1993, 1995; Chen et al. 2010). Therefore, how to properly model the dynamic process of US housing price and price the MI contracts is an important issue. Chen et al. (2010) show empirical evidence of the jump magnitude phenomenon when using US monthly national average new home data. In contrast to the assumption of lognormality in the jump magnitude generally made in previous literature, we would like to investigate whether the jump risk is symmetric or asymmetric and how the asymmetric jump risk affects the value of MI premiums if the jump risk of US housing prices exists to a significant extent. Furthermore, in view of the rising foreclosure rates of the borrowers and the mortgage insurers' huge losses, the default risk of the mortgage insurer has drawn more attention to the valuation of MI contracts, especially during the subprime mortgage crisis. Therefore, it is also vitally important to assess the impact of default risk of mortgage insurer on the MI premiums.

In addition to the interest rate change, the change in housing prices plays a crucial role in the pricing of MI contracts. In the previous literature, housing price change is assumed to follow a traditional Black-Scholes model (BSM), and this assumption is reasonable for relatively stable housing prices (e.g., Kau et al. 1992, 1993, 1995; Kau and Keenan 1995, 1999; Bardhan et al. 2006). Kau and Keenan (1996) use a compound Poisson process to only consider the down-jump component of housing prices in the case of catastrophic events. Chen et al. (2010) assume that the housing price process follows the log-normally distributed jump diffusion (LJD) process, capturing important characteristics of abnormal shock events. This assumption is consistent with the empirical observation of the US monthly national average of new home returns from January 1986 to June 2008. However, the traditional lognormality assumption involves a "generic jump" whose magnitude fluctuates between minus one and infinity, thus allowing the generation of both downward and upward jumps. Although the lognormal distribution has many useful properties, one drawback of this approach is the constraining of upward jumps and downward jumps to both come from the same distribution as well as the lack of precise differentiation between the probabilities of the occurrence of each type of jump.

Figure 1 shows the US national average of all home price returns for single-family mortgages from January 1986 to October 2008. There were four occasions when the monthly housing price changed by more than two standard errors per month. Furthermore, it can be seen that there were nine occasions when the monthly housing price changed by less than two standard errors per month. Therefore, the US national average of all home price returns seems to have the properties of excess kurtosis, skewness and asymmetric jump phenomenon. The excess kurtosis and skewness of the housing price change can be partially explained through the modeling of jumps and also through the use of stochastic volatility (Heston 1993) or both as shown in Pan (2002) or

Eraker et al. (2003).[1] However, stochastic volatility is not considered here to allow a focus on the effects of jumps alone. This paper concentrates on extending the previous studies to relax the assumption of a common lognormal jump distribution spanning both upward and downward jumps by allowing each jump type to possess distinct distributional properties. Because the liquidity of the real estate market is lower than that of the financial market, the jumps are rare events, i.e., crushes and large drawdowns. This study uses the asymmetric DEJD process, a type of finite-activity Lévy process, to capture asymmetric jump characteristics related to good and bad shock events that influence housing price.[2] These good and bad shock events, as Kau and Keenan (1996) and Chen et al. (2010) define them, can be financial (e.g., sudden severe upward and downward jumps in housing prices due to favorable or unfavorable economic news for the local economy, such as announcements regarding expansionary or contractionary monetary policy). In general, downward jumps in the housing price are more sensitive than upward jumps in the housing price based on the valuation of MI premiums. Therefore, the asymmetric jump risk for housing prices plays an important role in pricing for MI contracts, and the frequency and magnitude of the downward jumps in the housing price are particularly important. Some studies (e.g., Sutton (2002), Borio and Mcguire (2004), and Tsatsaronis and Zhu (2004)) indicate a significant negative relationship between the real interest rates and housing prices. We also incorporate the interest rate process into the dynamic process of housing price change to capture the interest-rate sensitivity of the rate of change of housing prices.

In view of the subprime mortgage crisis in the US, Mortgage Insurance Companies of America (MICA) reports that large mortgage insurers reported $2.6 billion in losses in 2008, sparking concerns that rising foreclosure rates of the borrowers could compel the industry into a money crisis. For instance, shares of Radian Guaranty, Triad, and PMI Mortgage Insurance Group lost 90 percent of their value in 2007; Triad Guaranty Insurance Corporation failed to meet capital requirement on March 31, 2008 and is even going out of business. MICA reports that Triad's risk-to-capital ratio, 27.7:1, exceeded the maximum (25:1) generally allowed by insurance regulations and Illinois insurance law. As we know, the default risk of the mortgage insurer is generally not considered by the previous pricing model of MI contracts.

There are three contributions to the pricing of MI contracts in this paper. First, we use US housing price data to find that the asymmetric DEJD process is the best fit by using the quasi-Newton algorithm, Bayesian information criterion (BIC) and

[1] Stochastic volatility usually has a larger impact on long-term options, whereas the presence of jumps mostly benefits the pricing of short-term near-the-money options.

[2] In general, there are two types of Lévy processes: the finite-activity Lévy process and the infinite-activity Lévy process. The finite-activity Lévy process generates only a finite number of jumps during any finite time interval. Examples of such models are the Merton jump-diffusion model with Gaussian jumps and the Kou model with asymmetric double exponential jumps. On the other hand, the infinite-activity Lévy model can generate an infinite number of small jumps at any finite time interval. Because the liquidity of the real estate market is lower than that of the financial market, the number of jumps should be finite. Furthermore, in finite-activity Lévy processes, the dynamic structure of the process is easy to understand and describe because the distribution of jump sizes is known. Such processes are also easy to simulate, and it is possible to use efficient Monte Carlo methods for pricing path-dependent options. Hence, this paper uses a finite-activity Lévy model to describe asymmetric jumps in the housing price process.

likelihood-ratio test (LR test). Next, to be consistent with the asymmetric jump behavior of US housing prices, the relationship between the interest rate and housing prices and mortgage insurers' default risk, this paper develops a contingent-claim framework for valuing an MI contract. We adopt a structural approach to model the default probability of the mortgage insurer. The mortgage insurer's total asset and liability value consists of two risk components: risk in interest rate and housing price. Finally, the sensitivity analysis examines how the asymmetric jump risk of housing prices and the default risk of the mortgage insurer impact the valuation of MI contracts and the default risk premium. We find that the shock frequency of the abnormal bad events has the most significant effect on the MI premium, and the asset-liability structure of the mortgage insurer and shock frequency of the abnormal bad events show the greatest effect of all parameters on the default risk premium. This implies that the insurer must carefully consider the impact of the shock frequency of the abnormal bad events when pricing the MI contracts.

The remainder of this paper is organized as follows. Section "Model" illustrates the model. Section "Valuation of Mortgage Insurance Contract" derives the pricing formulae for MI contracts under asymmetric DEJD. Empirical and numerical analyses are presented in Section "Empirical and Sensitivity Analysis". Section "Conclusions" summarizes the paper and gives conclusions.

Model

This study adopts a structural approach to model the default probability of the mortgage insurer. Because the interest rate, housing prices and the mortgage insurer's asset–liability structure specifications are crucial factors in determining the value of MI contracts, we assume that the interest rate, housing prices and the mortgage insurer's liability are related and that the interest rate and the mortgage insurer's assets are related. This section outlines the dynamic processes of the interest rate process, the borrower's housing price, the mortgage insurer's assets, and the mortgage insurer's liability under the risk-neutral measure Q.[3]

The Instantaneous Interest Rate Process

Following previous studies, (e.g., Kau et al. 1992, 1993, 1995), the instantaneous interest rate is assumed to follow the square-root process of Cox et al. (1985). Therefore, the interest rate process under the physical probability measure P is as follows:

$$dr(t) = \eta_r(\theta - r(t))dt + \nu\sqrt{r(t)}dW_r^P(t), \tag{1}$$

where η_r is the mean-reverting force measurement, θ denotes the long-run mean of the interest rate, ν presents the volatility parameter for the interest rate, and $W_r^P(t)$ is a Wiener process under the physical probability measure P. According to Girsanov's

[3] As argued by Bardhan et al. (2006), the valuation of MI can be also obtained if the assumption of the risk neutrality of agents is relaxed and insurance contracts are assumed to be traded. Therefore, we can price MI contracts without assuming the risk neutrality of the agents and instead assuming that MI contracts are traded.

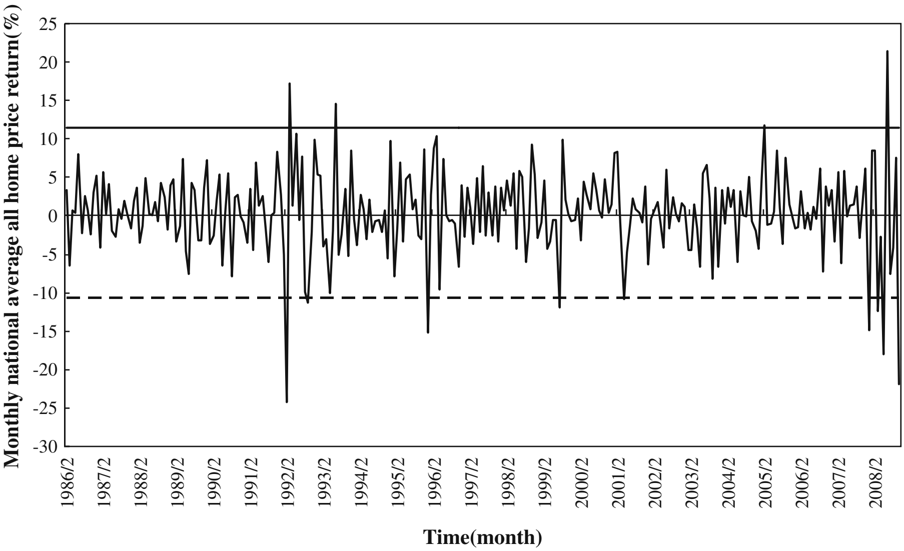

Fig. 1 US national average all home price returns for single-family mortgage. Note that the solid (dashed) line represents the mean of the housing price return plus (minus) two standard errors

theorem, $dW_r^Q(t) = dW_r^P(t) - \lambda_r\sqrt{r(t)}/v\,dt$, the interest rate process under the risk-neutral measure Q can be described as

$$dr(t) = \eta_r^*\left(\theta^* - r(t)\right)dt + v\sqrt{r(t)}dW_r^Q(t), \tag{2}$$

where the term λ_r is the market price of interest rate risk, and following Cox et al.'s (1985) assumption that λ_r is a constant and that $W_r^Q(t)$ is a Wiener process under the risk-neutral measure Q, $\eta_r^* = \eta_r - \lambda_r$, $\theta^* = \eta_r\theta/\eta_r - \lambda_r$.

The Housing Price Process

Kau and Keenan (1996) and Chen et al. (2010) use the LJD process to describe the change in housing prices. However, Fig. 1 seems to show that the change in US housing prices is asymmetric. Therefore, this study uses asymmetric DEJD to describe up-jump and down-jump components of the change in housing prices. Furthermore, previous studies (e.g., Harris (1989), Abraham and Hendershott (1996), Englund and Ioannides (1997), Sutton (2002), Borio and Mcguire (2004), and Tsatsaronis and Zhu (2004)) indicate a significant negative relationship between the real interest rates and housing prices. Therefore, this paper assumes that the change rate of housing price dynamics under the physical probability measure P is governed by the following process:

$$\frac{dH(t)}{H(t)} = \mu_H dt + \phi_{rH} dW_r^P(t) + \sigma_H dW_H^P(t) + d\sum_{n=0}^{N^P(t)}\left(V_n^P - 1\right), \tag{3}$$

where μ_H and $\sigma_H = \widetilde{\sigma}_H\sqrt{1-\rho_{rH}^2}$ are the drift and volatility terms of the rate of change rate housing prices, $\widetilde{\sigma}_H$ is the total volatility of the rate of change of housing prices and ρ_{rH} is the correlation coefficient of interest rates and housing price, $\phi_{rH} = \widetilde{\sigma}_H \rho_{rH}$ is negative and represents the instantaneous interest rate sensitivity of change rate of house price and $W_H^P(t)$ is a Wiener processes under the physical probability measure *P*. $N^P(t) = N_u(t) + N_d(t)$ is an independent Poisson process with intensity parameters $\lambda=\lambda_u+\lambda_d$, where λ_u and λ_d represent the intensity of up-jumps and down-jumps, respectively. V_n^P is the jump magnitude and a sequence of independent identically distributed nonnegative random variables such that $Y^P = \ln\left(V_n^P\right)$ has an asymmetric double exponential distribution with the density function:

$$f_{Y^P}(y) = p\eta_u e^{-\eta_u y} 1_{\{y \geq 0\}} + (1-p)\eta_d e^{\eta_d y} 1_{\{y<0\}},\ \eta_u > 1,\ \eta_d > 0. \tag{4}$$

Equation 4 represents the distribution of the logarithm of the jump magnitudes under the asymmetric DEJD, which has a jump intensity λ, and Y^P has an independent identically distributed mixture distribution of exponential (η_u) and exponential (η_d) with probabilities *p* and 1-*p*, respectively. According to the Esscher transform, as the following martingale condition is satisfied,[4]

$$r(t) = \mu_H + \phi_{rH} \frac{\lambda_r \sqrt{r(t)}}{v} - \sigma_H^2 h + \int_R (e^y - 1) e^{hy} v(dy), \tag{5}$$

where $v(dy) = \lambda f_{Y^P}(y)dy$ is a Levy measure of *Y*, and *h* is a real value, the change rate of housing price dynamics under the risk-neutral measure *Q* can be written as follows:

$$\frac{dH(t)}{H(t)} = \left[r(t) - \lambda^Q \kappa\right]dt + \phi_{rH} dW_r^Q(t) + \sigma_H dW_H^Q(t) + d\sum_{n=0}^{N^Q(t)} \left(V_n^Q - 1\right), \tag{6}$$

where

$$\lambda^Q = \lambda \times \left[\frac{p\eta_u}{\eta_u - h} + \frac{(1-p)\eta_d}{\eta_d + h}\right], \kappa = \frac{\hat{p}\,\hat{\eta}_u}{\hat{\eta}_u - 1} + \frac{\left(1-\hat{p}\right)\hat{\eta}_d}{\hat{\eta}_d + 1} - 1,\ \hat{\eta}_u = \eta_u - h,$$
$$\hat{\eta}_d = \eta_d + h, \hat{p} = \frac{p\eta_u(\eta_d + h)}{p\eta_u(\eta_d + h) + (1-p)\eta_d(\eta_u - h)},$$

and V_n^Q is the jump magnitude under *Q* such that $Y^Q = \ln\left(V_n^Q\right)$ has an asymmetric DEJD with the density function:

$$f_{Y^Q}(y) = \hat{p}\hat{\eta}_u e^{-\hat{\eta}_u y} 1_{\{y \geq 0\}} + \left(1 - \hat{p}\right)\hat{\eta}_d e^{\hat{\eta}_d y} 1_{\{y<0\}},\ \hat{\eta}_u > 1,\ \hat{\eta}_d > 0. \tag{7}$$

Focusing on the jump specification of the housing price in Eq. 7, four special cases can be delineated:

Case (1) Suppose that $\hat{\eta}_u = \hat{\eta}_d$ and $\hat{\lambda}_u = \hat{\lambda}_d$ (i.e.,$\hat{p} = 0.5$); then the distribution of jumps will be symmetrical with a higher peak and a positive kurtosis relative to normal.

Case (2) Suppose that $\hat{\eta}_u = \hat{\eta}_d$ and $\hat{\lambda}_u \neq \hat{\lambda}_d$ (i.e.,$\hat{p} \neq 0.5$); then, relative to the geometric Brownian motion, the distribution of the return of the housing price will be skewed and have excess kurtosis, and the relative size of $\hat{\lambda}_u$ and $\hat{\lambda}_d$ will lead to negative or positive skewness.

[4] The detailed description of the Esscher transform and similar detailed proof of the martingale condition can see Carr and Madan (1999).

Case (3) Suppose that $\hat{\eta}_u \neq \hat{\eta}_d$ and $\hat{\lambda}_u = \hat{\lambda}_d$; again, the resulting return of the housing price will be skewed and show excess kurtosis. However, the relative size of $\hat{\eta}_u$ and $\hat{\eta}_d$ will determine whether the distribution is negatively or positively skewed.

Case (4) Suppose that $\hat{\lambda}_u = \hat{\lambda}_d = 0$; again, the resulting return of the housing price will be reduced to the geometric Brownian motion.

The Mortgage Insurer's Liability Process

In the previous literature, the liability process follows a lognormal diffusion process, such as Cummins (1988). However, this modeling fails to particularly consider the impact of stochastic interest rates and housing prices. This shortcoming leads to the need to pay attention to modeling the liability value of the mortgage insurer, as falling house prices and rising interest rates are the accelerating factors for the catastrophic nature of MI. Therefore, extending Duan et al. (1995) and modeling the mortgage insurer's total liability value as consisting of two risk components, i.e., interest rate and house price risks, the change rate of the liability of the mortgage insurer under the physical probability measure P can be described as

$$\frac{dL(t)}{L(t)} = \mu_L dt + \phi_{rL} dW_r^P(t) + \phi_{HL} dW_H^P(t) + \sigma_L dW_L^P(t), \tag{8}$$

where μ_L and $\sigma_L = \sqrt{\tilde{\sigma}_L^2 - \phi_{rL}^2 - \phi_{HL}^2}$ are the drift and volatility terms of the rate of change of the liability value, and $\phi_{rL} = \tilde{\sigma}_L \rho_{rL}$ is the instantaneous interest rate sensitivity of the rate of change of the liability value, where $\tilde{\sigma}_L$ is the total volatility of the rate of change of the liability value, and ρ_{rL} is a correlation coefficient of the interest rate and the mortgage insurer's liability. $\phi_{HL} = \tilde{\sigma}_L(\rho_{HL} - \rho_{rH}\rho_{rL})/\sqrt{1-\rho_{rH}^2}$ is the instantaneous house price sensitivity of the rate of change of the liability, where ρ_{HL} is the correlation coefficient between the housing price and the mortgage insurer's liability. $W_L^P(t)$ is a Wiener process. As the following martingale condition is satisfied,

$$r(t) = \mu_L + \phi_{rL}\frac{\lambda_r\sqrt{r(t)}}{v} - \phi_{HL}\sigma_H h + \sigma_L \eta_L, \tag{9}$$

in a risk-neutral measure Q, the rate of change of the liability of the mortgage insurer is governed by the following process:

$$\begin{gathered} \frac{dL(t)}{L(t)} = r(t)dt + \phi_{rL} dW_r^Q(t) + \phi_{HL} dW_H^Q(t) + \sigma_L dW_L^Q(t), \\ dW_L^Q(t) = dW_L^P(t) - \eta_L dt, \end{gathered} \tag{10}$$

where the term η_L is the market price of the liability value, and the term $W_L^Q(t)$ is a Wiener process under the risk-neutral measure Q.

The Mortgage Insurer's Asset Process

In addition to the typical way of modeling the asset dynamics by assuming a lognormal diffusion process for the asset value, the model explicitly takes into account the impact of stochastic interest rates on the asset value. This is important for modeling the asset value of the mortgage insurer, as it is common for mortgage insurers to hold a large proportion of fixed-income assets in their portfolios. The change rate of the asset value of the mortgage insurer under the physical probability measure P can be written as follows:

$$\frac{dA(t)}{A(t)} = \mu_A dt + \phi_{rA} dW_r^P(t) + \sigma_A dW_A^P(t), \tag{11}$$

where μ_A and $\sigma_A = \sqrt{1-\rho_{rA}^2}\,\widetilde{\sigma}_A$ are the drift and volatility terms of the rate of change of the asset value, and $\phi_{rA} = \rho_{rA}\widetilde{\sigma}_A$ is the instantaneous interest-rate sensitivity of the rate of change of the asset value, where ρ_{rA} is a correlation coefficient of the interest rate and the mortgage insurer's asset, and $\widetilde{\sigma}_A$ is the total volatility of the rate of change of asset values. $W_A^P(t)$ is a Wiener process. As the following martingale condition is satisfied,

$$r(t) = \mu_A + \phi_{rA}\frac{\lambda_r\sqrt{r(t)}}{v} + \eta_A\sigma_A, \tag{12}$$

in a risk-neutral measure Q, the rate of change of the assets of the mortgage insurer can be described as follows:

$$\frac{dA(t)}{A(t)} = r(t)dt + \phi_{rA} dW_r^Q(t) + \sigma_A dW_A^Q(t), \tag{13}$$

$$dW_A^Q(t) = dW_A^P(t) - \eta_A dt, \tag{14}$$

where the term η_A is the market price of the asset value. $W_A^Q(t)$ is a Wiener process under the risk-neutral measure Q.

Valuation of Mortgage Insurance Contract

According to section above, we can know the risk-neutral dynamic processes of the interest rate, the housing price, the liability value and the asset value. Use of the four dynamic processes can lead to the valuation of the MI via discounting of the expected payoffs in the risk-neutral measure Q. At origination, t=0, the lender issues a T-year loan mortgage for the amount of $B(0)=L_R H(0)$. Let L_R be the initial loan-to-value ratio and $H(0)$ be the initial housing price. We assume that the mortgage loan has an adjusted interest rate y and that installments c are paid annually. Therefore, with no prepayment or default prior to time t, the owed loan balance $B(t)$ at time $0 \le t \le T$ is as follows:

$$B(t) = \frac{c}{y}\left(1 - \frac{1}{(1+y)^{T-t}}\right). \tag{15}$$

This equation shows that the unpaid loan balance is equal to the value of an ordinary annuity with an annual payment equal to c and a discount rate equal to the contract rate y. In addition, at time $t=0$, the mortgage insurer writes an MI contract that agrees to indemnify the lender if the borrower defaults.

We assume that each time interval is a year and that the borrower has the opportunity to default at these times. Considering the mortgage insurer's default risk, the losses of MI at time t, $LOSS_D(t)$, can be written as follows:

$$LOSS_D(t) = \begin{cases} L_C B(t) & if\, H(t) < (1-L_C)B(t)\, and\, A(t) \geq L(t) \\ \frac{L_C B(t) A(t)}{L(t)} & if\, H(t) < (1-L_C)B(t)\, and\, A(t) < L(t) \\ B(t) - H(t) & if\, (1-L_C)B(t) \leq H(t) < B(t)\, and\, A(t) \geq L(t) \\ \frac{(B(t)-H(t))A(t)}{L(t)} & if\, (1-L_C)B(t) \leq H(t) < B(t)\, and\, A(t) < L(t) \\ 0 & otherwise \end{cases} \quad (16)$$

where L_C denotes the coverage ratio. The first and third terms of Eq. 16 follow the setup of Bardhan et al. (2006) and are expressed as the loss of the mortgage insurer if the borrower defaults at time $0 \leq t \leq T$, whereas the mortgage insurer does not default during the remaining life of the MI. Thus, during the remaining life of the MI, the value of the mortgage insurer's total assets $A(t)$ is higher than the value of the mortgage insurer's total liability $L(t)$, and thus the mortgage insurer does not default. The second and fourth terms of this equation are expressed as the recovery loss of the MI if the mortgage insurer defaults at time $0 \leq t \leq T$, i.e., $A(t)$ is less than $L(t)$, and the borrower also defaults at time $0 \leq t \leq T$ during the remaining life of the MI. The recovery loss is equal to the original loss of the MI multiplied by the recovery rate, $A(t)/L(t)$. Therefore, Eq. 12 indicates that the MI contract embeds a portfolio of vulnerable American puts that may be exercised when the mortgage borrowers default and the contract is compulsory to be terminated in the case of the default of the mortgage insurers.

The present value of the loss, i.e., the expected loss to the insurer conditional on the borrower's default happening at time $t \in T$ and discounted back to the present time, $DL(t)$, can be described as follows:

$$DL(t) = E^Q\left[e^{-\int_t^T r(s)ds} LOSS_i(t)\right],\ i = ND\, or\, D. \quad (17)$$

Some special cases in Eq. 17 can be delineated:

(a) If $A(t)/L(t) \to \infty$ (i.e., the mortgage insurer would not default) and there is a constant interest rate, the closed-form solution of Eq. 17 using put-call parity is given by the following expressions:[5]

$$\begin{aligned} DL(t) = P(t, K_1) - P(t, K_2) = \frac{e^{-\alpha k_1}}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega k_1} \frac{e^{-rt}\phi_{\widetilde{H}(t)}(\omega - i(\alpha+1))}{\alpha^2+\alpha-\omega^2+i(2\alpha+1)\omega} d\omega \\ - \frac{e^{-\alpha k_2}}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega k_2} \frac{e^{-rt}\phi_{\widetilde{H}(t)}(\omega - i(\alpha+1))}{\alpha^2+\alpha-\omega^2+i(2\alpha+1)\omega} d\omega + (K_1 - K_2)e^{-rt}. \end{aligned} \quad (18)$$

where

$$\phi_{\widetilde{H}(t)}(\omega - i(\alpha+1))$$

[5] The pricing procedure can see Carr and Madan (1999).

$$= \exp\left\{ i(\omega - i(\alpha+1))\left(\ln H(0) + \left(r - \tfrac{1}{2}\sigma_H^2 - \lambda^Q \kappa\right)t\right) - \tfrac{1}{2}(\omega - i(\alpha+1))^2 \sigma_H^2 t \right\} \times$$
$$\exp\left\{ \lambda^Q t \left(\frac{\hat{p}\hat{\eta}_u}{\hat{\eta}_u - i(\omega - i(\alpha+1))} + \frac{(1-\hat{p})\hat{\eta}_d}{\hat{\eta}_d + i(\omega - i(\alpha+1))} - 1 \right) \right\},$$
$$k_1 \equiv \ln K_1 = \ln B(t),\ k_2 \equiv \ln K_2 = \ln(1 - L_C)B(t),\ \widetilde{H}(t) \equiv \ln H(t).$$

(b) If $A(t)/L(t) \rightarrow \infty$ and a constant interest rate and a lognormal jump component of the housing price exist, the closed-form solution of Eq. 17 reduces to the closed-form formula of Chen et al. (2010).

(c) If $A(t)/L(t) \rightarrow \infty$ and a constant interest rate and no jump component of housing prices exist, the closed-form solution of Eq. 17 reduces to the closed-form formula of Bardhan et al. (2006).

Because the housing price is independent of the unconditional probability of the borrower's defaulting, the MI premium (FPA) with an asymmetric jump risk is given by the following expression:

$$FPA = (1+q)\sum_{t=1}^{T} P(t)DL(t), \qquad (19)$$

where q represents the gross profit margin, and $P(t) = 1 - e^{-\lambda_b t}$, λ_b denotes the default frequency of the borrower. Equation 19 implies that FPA is calculated by $1+q$ multiples of the fair price, i.e., $\sum_{t=1}^{T} P(t)DL(t)$, which is the summation of a series of the loss amounts of the insurer if the borrower defaults in each year from the beginning to expiration. Therefore, the insurer can decide for each year the probability that the borrower will default rather than at only maturity.

Empirical and Sensitivity Analysis

Data and Empirical Results

Our data come from the Federal Housing Finance Agency and contain the term on conventional single-family mortgages and the monthly national average of all home prices in the US. We investigate the monthly average of the prices of all homes with adjustable-rate mortgages.[6] Our sample period is from January 1986 to October 2008, leading to 274 observations for each variable. We use the asymmetric DEJD process (see Eq. 3) to compare the model's fitness for the national average of single-

[6] In addition to ARM loans, FRM loans are also available for the FHFA. US national average all home price returns for single-family FRM loans also seem to feature the asymmetric jump phenomenon. It shows that there were four occasions when the monthly housing price changed by more than two standard errors per month. And then it can be seen that there were six occasions when the monthly housing price changed by less than two standard errors per month. Hence, the asymmetric jump phenomenon in ARM loans seems to be higher than one in FRM loans. Because that this paper focuses on the asymmetric jump phenomenon of US national average all home price returns, for simplification, in the empirical study, only US national average all home price returns for single-family ARM loans were used. In further research, it could compare the asymmetric jump phenomenon of ARM loans and FRM loans, and then investigate the impacts of their asymmetric jump behaviors on MI premiums.

family mortgaged home prices in the US with results of the LJD and BSM generally used in previous literatures.[7]

Before estimating the parameters of the asymmetric DEJD, LJD and BSM, it is necessary to perform unit root tests on the data series to determine whether it is a stationary time series. The Augmented Dickey-Fuller (ADF) unit root test, together with the descriptive statistics, is reported in Table 1. Based on the unit root tests, the null hypothesis of a unit root was rejected for US national average returns on single-family mortgages, indicating that the US national average returns is stationary. Furthermore, the descriptive statistics show that the US national average returns have negative skewness and fat tails, and the Jarque-Bera test indicates that the distribution is non-normal.

Table 2 reports the parameters estimated and their standard errors by using quasi-Newton algorithms,[8] the BIC values and LR test from the asymmetric DEJD, LJD and BSM based on the national average of single-family mortgaged home prices. From this table, we can find that the mean term of the rate of change of home prices, μ_H, is significant and positive and that the instantaneous interest-rate sensitivity of the rate of change of home prices, ϕ_{rH}, is significant and negative in the BSM, the LJD and the asymmetric DEJD. This result is consistent with the empirical findings of Borio and Mcguire (2004) and Tsatsaronis and Zhu (2004). Furthermore, the volatility term of the change rate of home prices, σ_H, is larger in the BSM than in the LJD and the asymmetric DEJD, which is reasonable because the LJD and the asymmetric DEJD capture an abnormal volatility in the jump size term in addition to the normal volatility. Considering the LJD parameters, it appears that a jump in the return process occurs approximately once every 2.773 years λ^{-1}. The average yearly jump size α is −0.065. The standard deviation of the yearly jump magnitudes β is 0.116. Turning to the asymmetric DEJD, it appears that the up-jump probability is 0.2799, and the t-statistic test rejects the null hypothesis of p=0.5 at a significance level of 1% (t=0.2799–0.5/0.04898=–4.494). This implies that the up-jump and down-jump probability distributions are asymmetric. Good news in return process occurs roughly once every 0.908 years $\left(\lambda_u^{-1}; \lambda_u = p \times \lambda\right)$, and bad news occurs roughly once every 0.353 years $\left(\lambda_d^{-1}; \lambda_d = (1-p) \times \lambda\right)$. The mean up-jump and down-jump magnitudes, in yearly percentages, are 3.98% and 4.42%, respectively, (η_u^{-1} and η_d^{-1}). Based on the jump events and the jump magnitudes of the return of US housing prices, the down-jump is significantly greater than the up-jump. Therefore, we can conclude that the distribution of the return of US housing prices will be negatively skewed and have excess kurtosis. Moreover, the model with the smallest BIC provides the best fit to the data. We know that the asymmetric DEJD is clearly superior to the BSM and LJD. Furthermore, the LR test also indicated that the LJD is superior to the BSM and that the asymmetric DEJD is superior to the LJD. That is, the asymmetric DEJD shows the best model fit for the US housing price returns.

[7] The limitation of using national average house prices is that, compared with individual house prices, national average house prices may reduce the level of volatility of housing prices and understate the degree of risk for mortgage insurers who do not operate nationwide.

[8] For a detailed description of the maximum-likelihood function of asymmetric DEJD, LJD and BSM, see Appendix A.

Table 1 Descriptive statistics and unit root tests

Panel A: Descriptive Statistics							
Mean	Median	Max.	Min.	Std. Dev.	Skew	Kurt	Jarque-Bera
0.00411	0.00420	0.21385	−0.24246	0.05526	−0.51617	5.90417	108.062*
Panel B: Augmented Dickey-Fuller test							
Intercept			Trend & Intercept		None		
−12.9175*			−12.9164*		−15.1943*		

Asterisks denote statistical significance at the 1% (*) level

Table 2 Maximum likelihood estimates and testing for the asymmetric DEJD, LJD, and BSM

Model	μ_H	σ_H	φ_{rH}	α	β	λ	η_u	η_d	p	BIC	LR
DEJD	$1.416\times10^{-1*}$	$1.402\times10^{-1*}$	$-4.948\times10^{-2*}$	–	–	3.933*	25.099*	22.647*	$2.799\times10^{-1*}$	−798.699*	L_{LJD}-L_{DEJD}=−3.4402*
	(9.617×10^{-3})	(3.006×10^{-3})	(2.094×10^{-3})*			(4.815×10^{-1})	(1.779)	(7.365×10^{-1})	(4.898×10^{-2})		
LJD	$8.795\times10^{-2*}$	$3.920\times10^{-2*}$	$-1.695\times10^{-1*}$	$-6.506\times10^{-2*}$	$1.159\times10^{-1*}$	$3.606\times10^{-1*}$	–	–	–	−797.428	L_{BSM}-L_{LJD}=−11.8878*
	(8.988×10^{-4})	(5.915×10^{-3})	(1.565×10^{-3})	(2.407×10^{-3})	(1.164×10^{-3})	(2.206×10^{-2})					
BSM	$6.768\times10^{-2*}$	$1.657\times10^{-1*}$	$-9.579\times10^{-2*}$	–	–	–	–	–	–	−790.481	–
	(1.463×10^{-3})	(2.278×10^{-3})	(3.956×10^{-3})								

DEJD: asymmetric double exponential jump diffusion

LJD: the log-normally distributed jump diffusion

BSM: Black-Scholes model

The values in parentheses denote standard errors

* is a significance level at 1%.

Sensitivity Analysis for Mortgage Insurance Premiums

Base Parameters and Value of Mortgage Insurance Premiums

The results in this section are based on a set of parameters from the data and estimation of the national average of all home price returns for single-family mortgages and show that the asymmetric DEJD is a good model. The base parameters and their standard errors of the normal volatility of the Brownian component (σ_H), the parameter of down-jump magnitudes (η_d) and the shock frequency of the abnormal bad events (λ_d) on the MI premium are taken from Table 2. Table 3 presents the effects of these base parameter values on the MI

Table 3 Base parameter values of mortgage insurance premium

Interest rate parameters		Values
r	Initial instantaneous interest rate	0.05
η_r	Magnitude of mean-reverting force	0.2
λ_r	Market price of interest rate risk	0.01
θ	Long-run mean of interest rate	0.1
v	Volatility of interest rate	0.05
Liability parameters		
L	Liability value of the mortgage insurer	10000000
φ_{rL}	Interest rate sensitivity of change rate of liability value	−0.1
φ_{HL}	House price sensitivity of change rate of liability value	−0.1
σ_L	Volatility terms of change rate of liability value	0.05
Asset parameters		
A	Asset value of the mortgage insurer	V/L=1.5, 1.75 and 2
σ_A	Volatility terms of change rate of asset value	0.05
φ_{rA}	Interest rate sensitivity of change rate of asset value	−0.1
Housing price parameters		
$H(0)$	Initial housing price	242300
φ_{rH}	Interest rate sensitivity of change rate of house price	−0.04948
σ_H	Volatility terms of change rate of house price	0.1402
η_u	The parameter of up-jump size	25.099
η_d	The parameter of down-jump size	22.647
λ	The parameter of jump intensity	3.933
p	The up-jump size probability	0.2799
Other parameters		
L_R	Initial loan-to-value ratio	0.85
L_C	Coverage ratio	0.3
c	Installments	16597
T	Term to maturity of mortgage contract	30 years
y	Contract interest rate of mortgages	0.07
q	Gross profit margin	0.05
λ_b	Default frequency of the borrower	0.05

premium. Based on the setup of Eq. 12, the MI contract embeds a portfolio of vulnerable American puts. This paper uses the Least-Squares Monte Carlo (LSM) algorithm provided by Longstaff and Schwartz (2001) to calculate the base value of the MI premium as 1,896.98 dollars by using Eqs. 12~13.

Parameters Values Matter: Sensitivity Analysis

Table 4 further reports the sensitivity analysis of the MI premiums. The indicated parameters plus (minus) two standard errors are used to demonstrate the normal volatility effect, the down-jump magnitude effect, the shock frequency of the abnormal bad-events effect, and the asset-liability structure of the mortgage insurer effect, respectively. From the sensitivity analysis, if the σ_H increases by two standard errors from 0.140 to 0.146 while all other parameters are fixed, the MI premium should be 1,959.09 dollars rather than 1,896.98 dollars. Therefore, the MI premium increases by 3.27%. On the other hand, if the σ_H is reduced by two standard errors from 0.140 to 0.134, the MI premium decreases to 1,837.94 dollars from 1,896.98 dollars, a reduction of 3.11%. Thus, the normal volatility is related positively to the MI premium. The parameters of the down-jump magnitude η_d are set to 21.174, 22.647, and 24.120. When η_d increases by two standard errors from 22.647 to 24.120, the MI premium is reduced from 1,896.98 dollars to 1,786.98 dollars. Thus, the MI premium decreases by 5.80%. This implies that if the η_d increases, the mean down-jump magnitude will decrease, the housing price will increase, and the MI premium will decrease. If the η_d is reduced by two standard errors from 22.647 to 21.174, the MI premium increases to 2,073.05 dollars from 1,896.98 dollars, a growth of 9.28%. In other words, the down-jump magnitudes vary negatively with the MI premium. The parameters of the shock frequency of the abnormal bad events λ_d are set to 2.139, 2.832, and 3.526. When the λ_d increases by two standard errors from 2.832 to 3.526 when all other parameters are fixed, the MI premium rises to 2,304.16 dollars from 1,896.98 dollars. Thus, the MI premium increases by 21.46%. This implies that when λ_d increases, the housing price decreases, and the MI premium increases. In other words, λ_d increases with the MI premium. On the other hand, if the λ_d is reduced by two standard errors from 2.832 to 2.139 when all other parameters are fixed, the MI premium decreases to 1,499.80 dollars from 1,896.98 dollars, a reduction of 20.94%. Thus, the shock frequency of the abnormal bad events is related positively to the MI premium. The parameters of the asset-liability structure of the mortgage insurer are set to 1.5, 1.75 and 2. As the asset-liability structure of the mortgage insurer increases 0.25 from 1.75 to 2, the MI premium should be 1,902.34 dollars rather than 1,896.98 dollars. Thus, the MI premium increases by 0.28%. This implies that if the asset-liability structure of the mortgage insurer increases, the default probability of the MI company will decrease, and MI premiums will increase. On the other hand, if the asset-liability structure of the mortgage insurer is reduced by 0.25 from 1.75 to 1.5, the MI premium decreases to 1,876.19 dollars from 1,896.98 dollars, representing a reduction of 1.10%. Therefore, the asset-liability structure of the mortgage insurer is positively related to the MI premium. In particular, λ_d has the largest effect of all parameters on the MI premium. This economic implication is that when a home owned by the borrower is mortgaged to the lender and the insurer writes a MI contract that promises to compensate the lender only when the borrower defaults, in addition to

Table 4 Sensitivity analysis of mortgage insurance premium

λ_d	η_d	σ_H	No default risk	Default risk					
				Asset-liability structure			Default risk premiums		
				1.5	1.75	2	1.5	1.75	2
2.139	21.174	0.134	1561.16	1516.61	1534.97	1540.72	44.55	26.19	20.44
			(−17.70%)	(−20.05%)	(−19.08%)	(−18.78%)	(26.68%)	(−25.52%)	(−41.86%)
		0.140	1629.66	1579.30	1598.01	1605.64	50.36	31.65	24.02
			(−14.09%)	(−16.75%)	(−15.76%)	(−15.36%)	(43.22%)	(−10.00%)	(−31.69%)
		0.146	1700.46	1643.39	1664.16	1671.64	57.07	36.31	28.83
			(−10.36%)	(−13.37%)	(−12.27%)	(−11.88%)	(62.29%)	(3.25%)	(−18.02%)
	22.647	0.134	1454.53	1418.54	1436.67	1441.99	35.99	17.86	12.54
			(−23.32%)	(−25.22%)	(−24.27%)	(−23.98%)	(2.36%)	(−49.22%)	(−64.34%)
		0.140	1525.61	1479.71	1499.80	1506.25	45.90	25.81	19.36
			(−19.58%)	(−22.00%)	(−20.94%)	(−20.60%)	(30.53%)	(−26.61%)	(−44.95%)
		0.146	1598.54	1545.19	1565.75	1573.83	53.35	32.79	24.72
			(−15.73%)	(−18.54%)	(−17.46%)	(−17.03%)	(51.72%)	(−6.75%)	(−29.71%)
	24.120	0.134	1340.15	1309.25	1324.73	1330.46	30.90	15.42	9.68
			(−29.35%)	(−30.98%)	(−30.17%)	(−29.86%)	(−12.13%)	(−56.15%)	(−72.47%)
		0.140	1413.71	1376.73	1394.52	1401.40	36.98	19.19	12.31
			(−25.48%)	(−27.43%)	(−26.49%)	(−26.12%)	(5.15%)	(−45.44%)	(−64.99%)
		0.146	1488.81	1444.23	1464.35	1470.55	44.58	24.46	18.26
			(−21.52%)	(−23.87%)	(−22.81%)	(−22.48%)	(26.78%)	(−30.43%)	(−48.07%)
2.832	21.174	0.134	2054.57	1995.48	2018.25	2025.68	59.09	36.32	28.88
			(8.31%)	(5.19%)	(6.39%)	(6.78%)	(68.02%)	(3.29%)	(−17.86%)
		0.140	2117.63	2051.36	2073.05	2080.06	66.27	44.58	37.58
			(11.63%)	(8.14%)	(9.28%)	(9.65%)	(88.46%)	(26.78%)	(6.86%)

		0.146	2180.26	2111.08	2133.82	2140.53	69.18	46.44	39.74
			(14.93%)	(11.29%)	(12.49%)	(12.84%)	(96.74%)	(32.05%)	(13.00%)
	22.647	0.134	1870.06	1820.42	1837.94	1843.54	49.64	32.12	26.51
			(−1.42%)	(−4.04%)	(−3.11%)	(−2.82%)	(41.16%)	(−8.67%)	(−24.60%)
		0.140	1932.14	1876.19	**1896.98**	1902.34	55.95	**35.17**	29.81
			(1.85%)	(−1.10%)	**(BVP)**	(0.28%)	(59.10%)	**(BVD)**	(−15.24%)
		0.146	1996.94	1938.48	1959.09	1966.47	58.46	37.85	30.47
			(5.27%)	(2.19%)	(3.27%)	(3.66%)	(66.24%)	(7.63%)	(−13.35%)
	24.120	0.134	1753.90	1704.26	1723.73	1730.00	49.64	30.17	23.90
			(−7.54%)	(−10.16%)	(−9.13%)	(−8.80%)	(41.15%)	(−14.22%)	(−32.03%)
		0.140	1820.92	1765.13	1786.98	1794.45	55.79	33.94	26.48
			(−4.01%)	(−6.95%)	(−5.80%)	(−5.40%)	(58.64%)	(−3.48%)	(−24.71%)
		0.146	1891.18	1827.75	1849.96	1858.38	63.43	41.22	32.80
			(−0.31%)	(−3.65%)	(−2.48%)	(−2.03%)	(80.37%)	(17.21%)	(−6.72%)
3.526	21.174	0.134	2480.07	2408.95	2431.88	2440.10	71.12	48.19	39.97
			(30.74%)	(26.99%)	(28.20%)	(28.63%)	(102.24%)	(37.04%)	(13.67%)
		0.140	2535.25	2458.83	2481.24	2490.41	76.43	54.01	44.85
			(33.65%)	(29.62%)	(30.80%)	(31.28%)	(117.33%)	(53.59%)	(27.53%)
		0.146	2591.34	2510.10	2534.85	2542.46	81.24	56.49	48.88
			(36.60%)	(32.32%)	(33.63%)	(34.03%)	(131.01%)	(60.65%)	(38.99%)
	22.647	0.134	2297.06	2228.21	2249.24	2257.38	68.85	47.82	39.68
			(21.09%)	(17.46%)	(18.57%)	(19.00%)	(95.79%)	(35.99%)	(12.84%)
		0.140	2350.02	2280.84	2304.16	2312.38	69.18	45.86	37.64
			(23.88%)	(20.24%)	(21.46%)	(21.90%)	(96.72%)	(30.41%)	(7.04%)
		0.146	2413.99	2336.88	2360.14	2368.40	77.11	53.85	45.59
			(27.25%)	(23.19%)	(24.42%)	(24.85%)	(119.26%)	(53.12%)	(29.63%)
	24.120	0.134	2144.86	2078.04	2097.23	2104.84	66.82	47.63	40.02
			(13.07%)	(9.54%)	(10.56%)	(10.96%)	(90.01%)	(35.43%)	(13.81%)
		0.140	2202.46	2131.42	2152.52	2160.54	71.04	49.94	41.91

Table 4 (continued)

λ_d	η_d	σ_H	No default risk	Default risk					
				Asset-liability structure			Default risk premiums		
				1.5	1.75	2	1.5	1.75	2
			(16.10%)	(12.36%)	(13.47%)	(13.89%)	(102.01%)	(42.02%)	(19.19%)
		0.146	2266.22	2188.37	2210.46	2217.96	77.86	55.76	48.26
			(19.46%)	(15.36%)	(16.53%)	(16.92%)	(121.39%)	(58.57%)	(37.23%)

BVP (BVD) denotes the base value of the MI premium (default risk premiums) and the values in parentheses denote the changing rate of the MI premium and default risk premiums the normal volatility of the housing price, the insurer must consider the impact of λ_d when pricing the MI contracts.

Table 4 also reports the sensitivity analysis of default risk premiums. The default risk premiums are computed as premiums without counterparty risk minus MI premiums with counterparty risk. We find that the default risk premiums are almost positively related to the normal volatility and the shock frequency of the abnormal bad events but negatively related to the asset-liability structure of the mortgage insurer. This table also indicates that the asset-liability structure of the mortgage insurer and the shock frequency of the abnormal bad events have the greatest effect of all parameters on the default risk premium. The economic implications are as follows: when the asset-liability structure of the mortgage insurer increases, the likelihood that the mortgage insurer will default decreases (i.e., the credit rating of the mortgage insurer increases), and thus, the default risk premium of the borrower decreases. Hence, the credit rating of the mortgage insurer could influence MI premiums. This also makes sense intuitively. Furthermore, in addition to the asset-liability structure of the mortgage insurer, no matter what we consider the default risk of mortgage insurer or not, the shock frequency of the abnormal bad events is a crucial factor when pricing MI contracts. Furthermore, Fig. 2 shows that the loan-to-value ratio and housing prices produce a wider range of MI premiums. These results are also consistent with those of previous studies such as Kau et al. (1992, 1993), Kau and Keenan (1995, 1996, 1999) and Chen et al. (2010). When the loan-to-value ratio (or housing price) is higher, the price of MI premiums is higher.

We further compare the extended model with the original Bardhan et al. (2006) model and to the Chen et al. (2010) model to provide more insight into how the asymmetric jump risk and the mortgage insurer's default risk influence MI premiums through changes in housing prices. Using the monthly observations reflecting US national average home prices for single-family mortgages as shown in Fig. 3, we depict the corresponding MI premiums from January 1987 to October 2009 for the three cases in Fig. 4. From Fig. 4, we find that each MI premium in our model is higher than in the Bardhan et al. (2006) model and lower than in the Chen et al. (2010) model. The difference between the Bardhan et al. (2006) model and our model is in

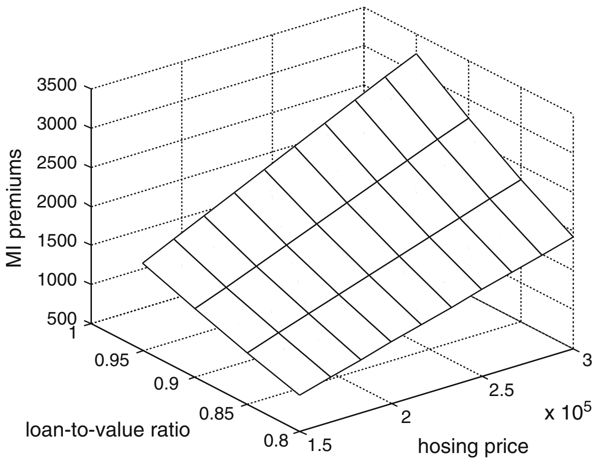

Fig. 2 The relationship between the loan-to-value ratio (housing price) and MI premiums

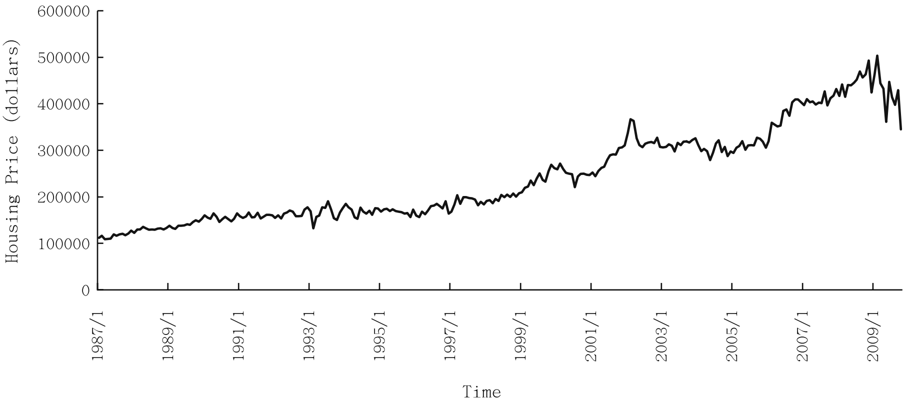

Fig. 3 US national average all home price for single-family mortgage

the range of $550–$2500 dollars. In particular, a higher housing price leads to a greater difference in MI premiums. Consequently, ignoring the impact of asymmetric jump risk on pricing for MI contracts would lead to severe underpricing for MI premiums, particularly given higher housing prices. On the other hand, the numerical results demonstrate that the difference between the MI premiums in the Chen et al. (2010) model (which does not consider the mortgage insurer's default risk) and our model (which does consider the mortgage insurer's default risk) is that default risk premiums for the mortgage insurer range from $28 to $110 dollars. Hence, ignoring the impact of default risk for the mortgage insurer on pricing MI contracts would lead to severe overpricing for MI premiums, particularly in the case of higher housing prices. These results make it intuitive that, when the mortgage insurer defaults, the mortgage insurer will only make a partial payment to the lender. Hence, based on the actuarial principal that the discount value of the expected loss equals the discount value of the expected revenue, the mortgage insurer will charge lower MI premiums. In practice, there is a significant relationship between the credit rating of a mortgage insurer and the MI premium. If the likelihood that the mortgage insurer will default

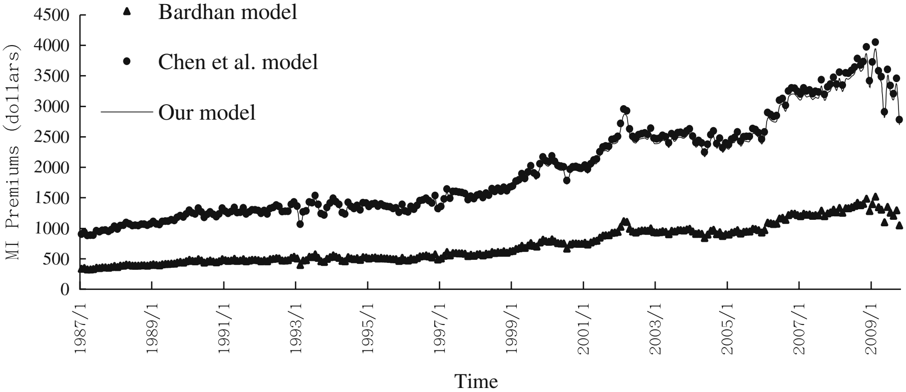

Fig. 4 The comparisons of MI premiums by different model

decreases (i.e., if the credit rating of the mortgage insurer increases), the mortgage insurer will have the capacity to charge higher MI premiums. On the contrary, if the credit rating of the mortgage insurer has been downgraded, the mortgage insurer should reduce its MI premiums. Consequently, we conclude that it is necessary to consider the asymmetric jump risk and the mortgage insurer's default risk when pricing the MI contract, particularly for higher housing prices.

Conclusions

The collapse of the subprime mortgage market and the mortgage insurers' massivelosses has drawn more attention to the precise valuation of MI contracts. This study derives the formula for MI contracts considering the asymmetric DEJD and mortgage insurer's default risk.

Furthermore, we use the national average of all single-family mortgaged home prices from January 1986 to October 2008 to estimate and test the asymmetric DEJD, the LJD and the BSM. The empirical results indicate that the DEJD is the best model to fit the national average of all single-family mortgaged home prices in the US. Finally, the sensitivity results show that the MI premium is an increasing function of the normal volatility, the mean down-jump magnitude, the shock frequency of abnormal bad events and the asset-liability structure of the mortgage insurer. Compared with the base valuation, when a housing crash occurs in the future, and if this crash causes the normal volatility σ_H to increase by two standard errors while all other parameters are fixed, the MI premium increases by 3.27%. Conversely, as the normal volatility decreases by two standard errors, the MI premium decreases by 3.11%. Furthermore, when the parameter of down-jump magnitudes, η_d, increases by two standard errors, the MI premium decreases by 5.80%. Conversely, as the parameter of down-jump magnitudes decreases by two standard errors, the MI premium should increase by 9.28%. Furthermore, when the shock frequency of the abnormal bad events, λ_d, increases by two standard errors, the MI premium increases by 21.46%. Similarly, the MI premium decreases by 20.94% if the shock frequency of the abnormal bad events decreases by two standard errors. Furthermore, as the asset-liability structure of the mortgage insurer increases by 0.25 standard errors, the MI premium increases by 0.28%. Conversely, if the asset-liability structure of the mortgage insurer is reduced by 0.25 standard errors, the MI premium decreases by 1.10%. Therefore, the shock frequency of the abnormal bad events has the most significant effect on the MI premium. Furthermore, we find that the asset-liability structure of the mortgage insurer and the shock frequency of the abnormal bad events show the greatest effect of all the parameters on the default risk premium. This implies that the insurer must carefully consider the impact of the shock frequency of abnormal bad events when pricing the MI contracts. Compare to Bardhan et al. (2006) model and Chen et al. (2010) model, we conclude that it is necessary to consider the asymmetric jump risk and mortgage insurer's default risk when pricing the MI contract, particularly for higher housing prices. Other potential improvements and possible extensions are given. First, in addition to the jump diffusion process, the positive serial correlation of US housing-price movements is a relevant and significant issue (see Case and Shiller 1989). Additionally, the structured change in US housing prices is also an important issue when pricing MI contracts. Many studies discussed the potential structural change in housing prices by employing the threshold regression, Markov-switching and smooth-threshold autoregressive (STAR) models, among others.

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Appendix A

Maximum Likelihood Function of Asymmetric DEJD, LJD and BSM

Let $H = \{H(0),H(1),H(2),\ldots,H(N)\}$ denote the realizations of housing price at equally-spaced times $t = 0,1,2,\ldots,N$. The one period rate of return $R_H(t) = 1\text{n}\, H(t)$ - $1\text{n}\, H(t\text{-}1)$ is IID. $\mathrm{fr}_k = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{-i\omega\left(R_{H,k}\right)}\phi_{R_{H,k}}(\omega)\,d\omega$, where k represents the asymmetric DEJD, LJD, or BSM, represents the density of return of housing price based on different model. And the characteristic function of return of housing price, $\phi_{R_{H,DEJD}}(\omega)$, under the asymmetric DEJD is given by:

$$\phi_{R_{H,DEJD}}(\omega) = \exp\left\{t\left(i\omega\widetilde{\mu}_H - \frac{1}{2}\omega^2\widetilde{\sigma}_H^2 + \lambda^P\left(\frac{p\eta_u}{\eta_u - i\omega} + \frac{(1-p)\eta_d}{\eta_d + i\omega} - 1\right)\right)\right\} \quad \text{(A.1)}$$

where $\widetilde{\mu}_H = \mu_H - \frac{1}{2}\sigma_H^2 - \frac{1}{2}\phi_{rH}^2$, $\widetilde{\sigma}_H^2 = \sigma_H^2 + \phi_{rH}^2$, and the characteristic function of return of housing price,$\phi_{R_{H,LJD}}(\omega)$, under the LJD is given by:

$$\phi_{R_{H,LJD}}(\omega) = \exp\left\{t\left(i\omega\widetilde{\mu}_H - \frac{1}{2}\omega^2\widetilde{\sigma}_H^2 + \lambda^P\left(\exp\left(i\,\omega\,\alpha - \frac{1}{2}\omega^2\beta^2\right) - 1\right)\right)\right\} \quad \text{(A.2)}$$

where α is the mean of jump size and β is the standard deviation of jump magnitudes. And the characteristic function of return of housing price, $\phi_{R_{H,BSM}}(\omega)$, under the BSM is given by:

$$\phi_{R_{H,BSM}}(\omega) = \exp\left\{t\left(i\omega\,\widetilde{\mu}_H - \frac{1}{2}\omega^2\widetilde{\sigma}_H^2\right)\right\}. \quad \text{(A.3)}$$

Denote k as the asymmetric DEJD, LJD, or BSM and suppose the k model, M_k, has parameter vector θ_k, where θ_k consists of j_k independent parameters to be estimated. Denote $\widehat{\theta}_k$ as the MLE of θ_k. The parameter space of the asymmetric DEJD, LJD, and BSM is denoted as $\theta_{DEJD} = (\mu_H, \sigma_H, \phi_{rH}, \lambda\ \eta_u, \eta_d, p)$, $\theta_{LJD} = (\mu_H, \sigma_H, \phi_{rH}, \lambda, \alpha, \beta)$, and $\theta_{BSM} = (\mu_H, \sigma_H, \phi_{rH})$, respectively. Hence, log-likelihood function based on different model is given by the following expression:

$$\log f(H|\theta_k, M_k) = \sum_{t=1}^{N} \ln(\mathrm{fr}_k(\mathrm{t})). \quad \text{(A.4)}$$

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