

Early Warning System using AVaR - Time Series Models with Standard Lévy Process

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Abstract

Empirical studies based on GARCH models show that the hypothesis that the distribution of residuals is normally distributed is often rejected (e.g., Duan, 1999; Menn and Rachev, 2009; Kim et al., 2010). Hence, most of the recent literature consider non-normal stock price models (e.g., Rachev et al., 2007; Fusai and Meucci, 2008; Farinelli et al., 2008; Sorwar and Dowd, forthcoming). Duan et al. (2006) enhance the classical GARCH model by adding jumps to the innovation process. This paper attempts to forecast both extreme events and highly volatile markets. We use in predicting real-world market crashed, such as Black Monday (October 19, 1987), the global economic meltdown attributable to the subprime mortgage meltdown in 2007 and the Lehman Brothers failure in the latter half year of 2008. Due to the properties of skewness, leptokurtosis, fat tails as well as the time varying volatility of ARMA-GARCH model with standard Lévy process, we compute VaR and AVaR and then develop early warning system and fear index based on ARMA-GARCH model with standard Lévy process. Empirical study, estimating VaR and AVaR for ARMA-GARCH model with standard Lévy process can be regarded as a suitable early warning system and the fear index, especially in the larger volatility clustering events of the FTSE, the HSI, the Nikkei 225, the DJIA, the NYSE, and the SPX.

Keywords: ARMA-GARCH, Lévy process, AVaR, fear index, extreme events

使用平均風險值建構提前預警機制-結合時間序列與 Lévy 過程

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摘要

過去以 GARCH 模型為基礎的實證研究顯示，虛無假設為常態分配的殘差通常是被拒絕(例如: Duan, 1999; Menn and Rachev, 2009; Kim et al., 2010)。因此，最近大多數文獻考慮非常態分配的股票價格模型(例如: Rachev et al., 2007; Fusai and Meucci, 2008; Farinelli et al., 2008; Sorwar and Dowd, forthcoming)。Duan et al.(2006) 加入跳躍創新的過程於古典 GARCH 模型。本文利用 ARMA-GARCH 模型與殘差服從 Lévy 過程以捕捉偏態、峰態、厚尾以及時間序列波動性的特性，並試圖預測極端事件和高度波動的股價指數市場，例如:黑色星期一 (1987 年 10 月 19 日)，和全球經濟危機之 2007 年次級抵押貸款危機和 2008 年下半年雷曼兄弟倒閉事件，計算 VaR 和 AVaR 以建立提前預警機制和恐慌指數。實證結果顯示，利用 ARMA-GARCH 模型於 Lévy 過程估計的 AVaR 可視為一個合理的提早預警機制和恐慌指數，尤其是英國倫敦金融時報 100 指數、香港恆生指數、日經 225 指數、道瓊指數、紐約證券交易所指數和 S&P500 指數的大波動群聚市場指數。

關鍵詞: ARMA-GARCH 模型、Lévy 過程、AVaR、恐慌指數、極端事件

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