

Portfolio Optimization and Conditional VaR by Using Generalized Hyperbolic Distributions

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Abstract

Regular portfolio optimization describes the dynamics of financial instruments' price with multivariate normal distribution. Without considering the skewness and kurtosis of assets return, optimization with multivariate normal distribution will underestimate the risk of a portfolio. This paper measures portfolio optimization and conditional Credit at Value (VaR) by using portfolio optimization approach of Rockafellar and Uryasev (2000) and Generalized Hyperbolic distribution (GH), and compares portfolio optimization and conditional VaR with multivariate Wiener process, Student's t distribution and NIG process. In the experiment, we calculate the optimal hedge positions of Dow Jones Industrial Average (DJIA), Financial Times Stock Exchange 100 Index (FTSE 100 Index) and Nikkei 225 (N225) with the performance function CVaR.

The empirical results show that portfolio optimization with multivariate NIG process has better fit than multivariate Wiener process and Student's t distribution. Therefore, we take sensitivity analysis for multivariate NIG process. Results indicate FTSE 100 and N225 in sensitivity analysis for CVaR has significant variation based on volatility and kurtosis parameters. Finally, we forecast the average potential risk of a portfolio based on the different time period by moving windows method, and the findings indicate that CVaR under multivariate NIG process is likely to downward phenomenon while forecasting the time period is getting longer.

Key Words: Portfolio optimization, Conditional VaR, Multivariate NIG process

使用 GH 分配之最適投資組合與條件風險值

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摘要

一般的最適投資組合模型皆假設金融商品股價波動服從多維常態分配。而在此模型假設下，不考慮資產的偏態與峰態，導致投資組合的風險常會有被低估的現象。所以，本文將以美國道瓊工業平均指數(DJIA)、英國富時 100 指數(FTSE 100)與日經 225 指數(N225)作為資產配置之標的資產，利用 Rockafellar and Uryasev (2000)之最適權重配置方法與考慮資產報酬率服從 Generalized Hyperbolic distribution (GH) 模型之下，估計最適投資組合以及條件風險值，並且分別比較在多維度常態、Student's t 與 NIG 模型下之最適投資組合與條件風險值。

實證結果顯示，多維度 NIG 模型之最適資產配置會優於多維度常態模型與多維度 Student's t 模型。因此，本文將針對多維度 NIG 模型進行敏感度分析，結果顯示當資產為英國富時 100 指數與日經 225 指數時，模型中的變異數與峰態參數變動，對投資組合之條件風險值的影響程度為最大。最後，探討當投資組合持有期間的不同時，樣本外條件風險值的變動情形；為了使模型預測更加穩健，採用移動視窗進行實證分析，結果顯示在多維度 NIG 模型之下，隨著持有期間越長，所提列的條件風險值似乎有一個降低的現象。

關鍵詞：最適投資組合、條件風險值、多維度 NIG 模型

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