

VIX 指數之 Levy 模型最適化估計與預測及 VIX 衍生性商品之定價

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摘要

有鑑於次貸風暴危機，此論文對 VIX 指數之跳躍風險進行分析與研究。我們使用不同 Lévy 模型捕捉 VIX 動態跳躍過程，並且藉由 1990/1/2 至 2009/12/31 期間之 VIX 指數資料進行實證分析，以驗證 VIX 指數是否有顯著之跳躍現象及探索最配適 VIX 指數動態過程之 Lévy 模型。接著，經由最配適 VIX 指數之 Lévy 模型進行 2010/1/4 至 2010/3/31 期間之 VIX 指數動態過程樣本外預測，進而分析其預測能力優劣。最後，藉由最配適 VIX 指數之 Lévy 模型進行 Esscher 轉換與傅立葉轉換對 VIX 衍生性金融商品進行訂價並且探討模型重要參數如何影響 VIX 衍生性金融商品之定價。實證結果發現 VIX 指數確實具有顯著之跳躍現象，NIG 模型最配適 1990/1/2 至 2009/12/31 期間之 VIX 指數動態過程，但其 2010/1/4~2010/3/31 之 VIX 指數動態過程樣本外預測卻為 LJD 模型具有較佳之預測能力，在數值分析方面，NIG 模型的厚尾參數對其 VIX 買權價格具有最大且負相關之影響，而 LJD 模型跳躍頻率與跳躍大小的波動度參數對其 VIX 買權價格具有最大且正相關之影響。

關鍵字: Lévy、VIX 衍生性商品、傅立葉轉換、Esscher 轉換、跳躍檢定。

Optimal Estimation and Forecasting of Lévy Models of VIX Index and Valuation of VIX Derivatives

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Abstract

In view of the subprime mortgage crisis, this article analyzes and investigates the jump risk of VIX index. We use different Lévy models to capture the dynamic jumps processes of VIX index and follow Bollerslev, Law and Tauchen (2007) to analyze the statistical significance of the jump risk. The more suitable Lévy models of VIX index are explored by using the data of VIX index from 1990/1/2~2009/12/31 in empirical analysis. Then we proceed with forecasting of VIX index from 2010/1/4~2010/3/31 by using the more suitable Lévy models of VIX index and analyze the ability of forecasting. Eventually, we use the more suitable Lévy models to price the VIX derivatives by utilizing Esscher transform and Fourier transform. Furthermore, we investigate how some important parameters of models impact on the value of VIX derivatives. The empirical results show that the dynamic processes of VIX index has significant jump phenomenon indeed. Besides, the NIG process is the fittest Lévy model of dynamic processes of VIX index from 1990/1/2~2009/12/31. But, the LJD model has more better ability of forecasting on forecasting dynamic processes of VIX index from 2010/1/4~2010/3/31. In numerical analysis, the parameter of the tail heaviness of steepness has the largest effect and negative correlation on the price of VIX call option of NIG process. On the other hand, the parameters of the jump intensity, and the standard deviation of total jump size have the largest effect and positive correlation on the price of VIX call option of LJD process.

Keywords: Lévy, VIX derivatives, Fourier transform, Esscher transform, jump test.

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