

# 共變異數矩陣估計與最適投資組合績效

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## 摘要

由於先前 Markowitz (1952)所提出的「平均數-變異數投資組合模型」(Mean-variance model)，會產生樣本誤差問題。因此，本文採用全域最小變異數模型、最小追蹤誤差方法(Minimum Tracking Error Variance)及共整合法來測試台灣股票市場的績效表現是否優於以台灣五十指數做為標竿指數之績效。並進一步比較三種方法績效之優劣。本研究的樣本期間為 1998 年 7 月開始，以 60 個月的資料當作移動視窗並進行月頻率替換至 2009 年 12 月為止。實證結果顯示，在考量風險的情況下，以全域最小變異模型所建構之投資組合風險為最低。而在績效表現的部份，則是以共整合方法所建構之投資組合最好，尤其是以 50 支股票所組成的投資組合績效表現最優，甚至超越了以台灣加權股價指數為標竿指數之績效。另外，最小追蹤誤差法在績效方面優於全域最小變異模型，而在風險部份又遠低於共整合法，因此就整體而言，以最小追蹤誤差法所建構之投資組合是優於其他兩種方法的。

關鍵詞：投資組合績效、全域最小變異模型、最小追蹤誤差方法、共整合方法

# **The Estimation of Covariance Matrix and the Performance of Optimal Portfolio**

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## **ABSTRACT**

The mean-variance model that Markowitz (1952) proposed has a sample error problem. This paper employs global minimum variance model, minimum tracking error method, and co-integration approach to investigate whether the performance is superior to the performance of Taiwan 50 index and further compare three methods' performance. The sample period will start in July 1998 to December 2009, and use rolling windows to replace the frequency. This research shows that the global minimum variance model has the lowest risk, and the co-integration method has the highest performance. However, the performance of minimum tracking error method is better than the global minimum variance model and lower volatility than the co-integration method. Therefore, the minimum tracking error method is the best method to construct the portfolio.

Keywords: portfolio performance, global minimum variance, minimum tracking error variance, co-integration approach

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